## ECEN 5713 Linear Systems Spring 2009 Midterm Exam \#2



Choose any four out of five problems.
Please specify which four listed below to be graded:
1)___ ; 2) ___ ; 3)___ ; 4) ___ ;

Name : $\qquad$

E-Mail Address:

## Problem 1:

Find the matrices $A, B, C$, and $D$ of state space representation for a composite configurations using two subsystems $\left\{A_{i}, B_{i}, C_{i}, D_{i}\right\}, i=1,2$, connected in negative feedback, with $\left\{A_{1}, B_{1}, C_{1}, D_{1}\right\}$ in the forward loop and $\left\{A_{2}, B_{2}, C_{2}, D_{2}\right\}$ in the feedback loop. Assume subsystem 1 (denoted by $\left.H_{1}\right),\left\{A_{1}, B_{1}, C_{1}, D_{1}\right\}$ has the transfer function $H_{1}(s)=\frac{s+5}{s^{2}+3 s+6}$, and subsystem 2 (denoted by $\left.H_{2}\right),\left\{A_{2}, B_{2}, C_{2}, D_{2}\right\}$ has the transfer function $H_{2}(s)=\frac{s+2}{s^{2}+4 s+3}$.


## Problem 2:

Let

$$
V^{\perp}=\operatorname{Span}\left(\left[\begin{array}{ccc}
1 & -2 & 1 \\
-1 & 3 & 0
\end{array}\right],\left[\begin{array}{ccc}
0 & 1 & -1 \\
2 & 0 & 0
\end{array}\right],\left[\begin{array}{ccc}
1 & -1 & 0 \\
1 & 3 & 0
\end{array}\right]\right),
$$

determine the original space, $V$. For $x=\left[\begin{array}{ccc}1 & -1 & 0 \\ 1 & 3 & 0\end{array}\right]$, find its direct sum representation of $x=x_{1} \oplus x_{2}$, such that $x_{1} \in V$, and $x_{2} \in V^{\perp}$ (i.e., the direct sum of spaces $V$ and $V^{\perp}$ is the set of all $2 \times 3$ matrics with real coefficients).

## Problem 3:

$$
A=\left[\begin{array}{lllll}
1 & 2 & 3 & 4 & 5 \\
2 & 3 & 4 & 1 & 2 \\
3 & 4 & 5 & 0 & 0
\end{array}\right]
$$

What are the rank and nullity of the above linear operator, $A$ ? And find the bases of the range spaces and the null spaces of the operator, $A$ ?

## Problem 4:

Consider the subspace of $\mathfrak{R}^{4}$ consisting of all $4 \times 1$ column vector $x=\left[\begin{array}{llll}x_{1} & x_{2} & x_{3} & x_{4}\end{array}\right]^{T}$ with constraints $x_{1}-2 x_{2}+3 x_{3}=0$ and $-x_{1}+2 x_{2}-3 x_{3}=0$. Extend the following set to form a basis for THE subspace:
$\left[\begin{array}{l}1 \\ 2 \\ 1 \\ 0\end{array}\right],\left[\begin{array}{c}-1 \\ -2 \\ -1 \\ 0\end{array}\right]$.

## Problem 5:

Show that

$$
\left[\begin{array}{c}
2 \\
-1 \\
1
\end{array}\right],\left[\begin{array}{c}
1 \\
2 \\
-3
\end{array}\right] \text { and }\left[\begin{array}{c}
3 \\
1 \\
-2
\end{array}\right],\left[\begin{array}{c}
-1 \\
3 \\
-4
\end{array}\right]
$$

span the same subspace $V$ of $\left(\mathfrak{R}^{3}, \mathfrak{R}\right)$.

